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BOOK REVIEW

A new history of greek mathematics, by Reviel Netz, Cambridge, Cambridge University Press, 2022, 540 pp., 58 b/w illus., $ 44.99 (hardcover), ISBN 9781108833844

In French, *haute vulgarisation* is a recognized genre. Not so in English, but even without the term *A New History of Greek Mathematics* is still an excellent representative. Regularly—and especially when complicated mathematical arguments unfold—the reader gets the impression of being in the presence of a spirited teacher in a lecture hall. Reviel Netz is a fine pedagogue.

A 'new' anything will easily be understood to pretend to replace the corresponding 'old.' Not so here. True, Netz states that 'I write this *History of Greek Mathematics* to replace that of Heath, written a century ago' (p. 457). But that is merely an argument for not exploring the history of Islamic mathematics in-depth (to which, none the less, 21 pages are dedicated). Instead, the reader is told that

> Thomas Little Heath's *History of Greek Mathematics*, published in 1921, has served as a reliable guide to many generations of scholars and curious readers. Historiographies went in and out of fashion, but Heath still stands, providing a clear and readable survey of the contents of most of the works of pure mathematics attested from Greek antiquity. […] I keep Heath by my side, and I urge you to do so as well. This new history does not aim to replace Heath's, and I do not aim at his encyclopedic coverage. (p. xi)

The perspectives of the two works are, indeed, almost orthogonal to each other (what one presents as a surface is reduced to a thin line in the other). Netz provides answers—sometimes solid, sometimes explicitly conjectural—to questions that were not asked by Heath, and rarely by anybody from his era. Heath's *History* was an internal and fairly exhaustive survey (not 'internalist'—Heath does no pretend to provide causal answers) while Netz altogether transcends the shallow internalist-externalist dichotomy. Though not encyclopaedic, Netz does present informative exemplary specimens of what a scholar like Asger Aaboe would speak about as 'the mathematics,' but always involved in its broader social and narrower intellectual context. Drawing on a small century's worth of work on social and cultural history, Netz is able to do so in a way which Otto Neugebauer in the late 1920s had to regret was as yet impossible, and which the intermediate generation (that of Aaboe) found uninteresting.

Netz's historiography, however, is not completely orthogonal to that of Heath. Heath, and almost all historians of mathematics until the 1960s (and many in later decades), believed in the late ancient, Neopythagorean narratives, those which ascribe the beginnings of Greek mathematics to Thales and Pythagoras. Netz, sources and source criticism at hand, discards these narratives (in the reviewer's opinion, convincingly).[1] Netz, on the other hand, has narratives of his own, often not based on but

[1]A warning to the reader: it is obviously easy to find convincing what one has also argued oneself – cf. Jens Høyrup, 'Hippocrates of Chios – His Elements and His Lunes: A Critique of Circular Reasoning', *AIMS Mathematics* 5 (2019), 158–84.

extrapolated from the sources—yet as a rule clearly set out to *be* such conjectural narratives.[2]

Quite solid, in the reviewer's opinion, and rather innovative (apart from being largely rooted in Netz's earlier work) is the periodization. The earliest phase (discussed in Chapter 2) – beginning with Archytas, Oinopides and Hippocrates of Chios and continuing until Theaitetos, Eudoxos and Menaichmos – is the one which creates the kind of mathematics we consider 'Greek': based on arguments around lettered diagrams, and often in dialogue with philosophy (that of Plato, but also the natural philosophy about the heavens).

The next creative phase (dealt with in Chapter 3) is that of Archimedes and those inspired by him – first of all Apollonios, active over a broad field even though little more than the *Conics* survives. Beyond Apollonios, Netz lists 32 figures who can be considered active mathematicians and whose activity can be ascribed (with varying degrees of certainty) to the generation after Archimedes. Before coming to Archimedes, Netz discusses in a 'prelude to Archimedes' the figures between the two creative phases – not least, evidently, Euclid, 'the inbetween mathematician' – together with the shift of geographical centre from Athens to Alexandria.

In its Hellenistic context, the generation of Archimedes had little to do with philosophy; the main aim of mathematical writing, as described by Netz, was to produce astounding results, neither to participate in a common endeavour nor to guide readers by the hand. One way was to present challenges without demonstration.

A third phase (Chapter 6) was that of 'canonization' and commentary, which was a regular feature of ancient high culture starting in the third century CE, often entangled with Neoplatonism. It shaped mathematics from the fourth century onward, where it is first represented by Pappos's *Collection* and perhaps by Diophantos's *Arithmetic* (which however could be considerably earlier), and further by Theon's redaction of the *Elements* (inter alia) and Hypatia's work on Diophantos. It also brought the Neopythagorean infatuation with numbers, and towards and beyond the end of Late Antiquity Proclos's commentary to *Elements* I, Simplicios's presentation of Hippocrates's work on the lunes, and Eutocios's commentaries to Archimedes – the latter leaving behind the link to Neopythagorean philosophy, perhaps because this philosophy (as Simplicios discovered at his costs) was becoming dangerous in an era of Christian theocracy. Gone was now the culture of mathematical surprise and challenges – a commentary to a venerated text may be critical but its purpose is to explain and thereby, in a certain way, to eliminate wonder. This final phase thus gave us not only the last link in the chain of manuscripts – those which Byzantine copyists would then use the ninth century onward – but also our image of 'Greek mathematics.'

Evidently, Netz deals with much more than these three creative or re-creative phases. Chapter 1, 'To the threshold of Greek mathematics,' offers a sensible presentation of ethnomathematics, and next takes up 'the invention of mathematics' in the context of the Inka, Mesopotamian and ancient Chinese states, followed by a closer study of the emergence of mathematics in Mesopotamia, and a delineation of what happened

[2]At times, admittedly (and as happens to all of us), what is first announced as a free conjecture, is later referred to as a fact. Thus, on page 495: "It is said that one of the reasons attracting Descartes to Leiden was the knowledge that Golius – a Renaissance scholar of Arabic – had there a manuscript with Apollonius translated into Arabic, with more of the *Conics* than were known in Greek." But on page 498 this becomes: "We are reminded […] of Descartes, relocating to Leiden so as to see the manuscript […] of Apollonius's lost Conics." The reader should thus remember which of Netz's "facts" are born as hypotheses. In the present case, the reader may find an account of the "facts" that is not fully consistent with Netz's hypothesis in Stephen Gaukroger, *Descartes: An Intellectual Biography* (Oxford: Clarendon Press, 1995) 187–211.

in the Greek world from the palace cultures of the Bronze Age until the early classical period (including what later Greek authors would believe and know about this period).

Chapter 4, 'mathematics in the world,' deals with the kinds of mathematics that were in broader or narrower *use* and/or taught as *paideia* to literate citizens. For the latter topic, the surviving papyri (mostly papyrus *fragments*) from Hellenistic and Roman Egypt serve as evidence.[3] It appears that most are teachers' copies, containing only statements and no demonstrations. In Netz's words, 'Euclid would be read in the same way as Homer was,' the schoolmasters teaching so much more Homer than mathematics (p. 233). Large sections of the chapter also deal with geography, military surveying, tactics, and writings on catapults and machines of marvel. As Netz points out, when higher mathematics (two mean proportionals, etc.) appears in such writings, their function seems first of all to demonstrate 'elite, literate sophistication' (p. 251). The end of the chapter takes up theoretical mechanics and the question of real versus symbolic mathematics.

Chapter 5, 'mathematics of the stars,' is even more pedagogical than the rest of the book. It starts with the kind of ethnoastronomy that is reflected in Hesiod's *Works and Days*, confronted with the 'bureaucrats' astronomy' which we know from Mesopotamia, and with Eudoxos's geometric model (where it is emphasized that we are not nearly as well-informed about it as often assumed). A thorough discussion (leading to no firm conclusion) deals with the possible roles of Archimedes (producer of a 'sphere,' a mechanical model), Apollonios and Hipparchos in the creation of the new epicycle-based astronomy and the adoption of Babylonian results. Hipparchos, and later Ptolemy (not only the *Almagest* but the whole *oeuvre*), are dealt with thoroughly.

Chapter 7, 'Into Modern science: The legacy of Greek mathematics,' is three-pronged. The first prong tells the story of preservation and repeated copying in Byzantium; the second, 'the world made from Baghdad,' speaks about the adoption and creative use of Greek mathematics in the Islamic world (to which, *in this respect*, even Latin Europe belongs after the '12th-century Renaissance'). The third, 'the Renaissance to end all renaissances,' finally, is a story about the influence of Greek mathematics in the European 17th century, *not least about the attempts to repair or restore what was perceived to have been lost*. This latter perspective is interesting and new; for the rest I shall refer to my sceptical response to Netz's earlier claims concerning the decisive role of Archimedes.[4]

Readers will probably find many occasions to disagree with Netz's conjectures or to find that they render only a small part of a complex truth – and many other points where they discover after second thoughts to agree or to have gained unexpected new insights. To go through all those points where it happened to the reviewer

[3]Even the Demotic Papyrus Cairo J.E. 89127-30, 89137-43 is drawn upon (p. 230) as evidence for what was taught to literate citizens in general. However, the papyrus in question is an obvious descendant from what had been taught to the scribes of the Pharaonic age (as we know it for instance from the Rhind Mathematical papyrus), though with certain matters shared with Mesopotamian texts prepared by the temple scribes in Seleucid times. Since the Ptolemaic rulers continued earlier administrative routines, the Demotic mathematical papyri are almost certainly linked to the education of scribes (public and temple officials), and therefore not informative about the *paideia* which better-class Greeks in general went through, not in Egypt and *a fortiori* not elsewhere.

The problem that is shown (to find the sides of a rectangle from its area and diagonal) is part of what was shared with the Mesopotamian tradition. Netz comments that now "we are back in Babylon. […] The key trick here, however, is somewhat closer to Pythagoras's theorem than is the case in extant Babylonian mathematics." This is a mistake. The same problem is solved in a similar way in the text Db_2-146 from early-18th-century Ešnunna, which even contains a proof building on the Pythagorean rule in partially abstract terms.

[4]See Jens Høyrup, "Where and How Did Archimedes Get In? Oblique and Labyrinthine Reflections," *Interdisciplinary Science Reviews* 47 (2022), 391–403. This is a response to Reviel Netz, "The Place of Archimedes in World History," *Interdisciplinary Science Reviews* 47 (2022), 301–330.

would ask for a triple essay review; renouncing that, I shall recommend that this speculative level of Netz's book be read as a stimulating invitation.

I shall, however, make two minor corrections. Firstly, in order to avoid restarting an ancient academic war: Jacques Sesiano did not discover the Arabic Diophantos, and never claimed to have done so (pp. 396*f*). Sesiano explains instead that 'in 1973, my thesis adviser, Gerald Toomer, learned of the existence of this manuscript in A. Gulchin-i Ma^{c}ānī's just-published catalogue of the mathematical manuscripts in the Mashhad Shrine Library, and secured a photographic copy of it'.[5]

Secondly, in order to avoid a false reconstruction of my own itinerary: My interpretation of Old Babylonian mathematical texts 'strictly as a historian of mathematics' did not precede their being put into the context of school and state administration (p. 25). That contextualization was actually my Mesopotamian starting point around 1975 (when I still believed in the Neugebauer–Thureau-Dangin interpretation), with the outcome first partially published in 1980.[6] It was only in 1981/82 that a question after a lecture about this sociological topic provoked me to undertake philological work.

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[5]Jacques Sesiano, (ed. and trans.), *Books IV to VII of Diophantus'* Arithmetica *in the Arabic Translation Attributed to Qusṭā ibn Lūqā* (New York: Springer, 1982), vii. Cf. also Jan P. Hogendijk, 'Review of Jacques Sesiano, Books IV to VII of Diophantus' *Arithmetica* in the Arabic Translation Attributed to Qusṭā ibn Lūqā. New York etc.: Springer, 1982', *Historia Mathematica* 12, 82–90.

[6]Jens Høyrup, 'Influences of Institutionalized Mathematics Teaching on the Development and Organization of Mathematical Thought in the Pre-Modern Period. Investigations into an Aspect of the Anthropology of Mathematics', *Materialien und Studien. Institut für Didaktik der Mathematik der Universität Bielefeld* 20 (1980), 7–137, 14–29. These results were soon after badly excerpted and pirated in Johan Fauvel and Jeremy Gray (eds.), *The History of Mathematics. A Reader*. (London: Macmillan, 1987), 43–5.